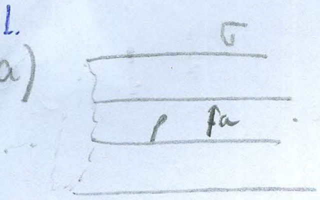
EVF 2015 1sem 17/03/2016

Q1.

a) 

$\sigma = -\frac{\rho a^2}{2b} \Rightarrow \sigma + v\rho = 0$

$r<a$:

$Q_{enc} = \pi r^2 \cdot l \cdot \rho$

$E = \frac{Q_{enc}}{2\pi r l \varepsilon_0} = \boxed{\frac{r \cdot \rho}{2\varepsilon_0 \cdot}}$

$a<r<b$

$E = \frac{Q_{enc}}{2\pi r l \cdot \varepsilon_0} \Rightarrow E = \frac{a^2 \rho}{2r\varepsilon_0 \cdot \varepsilon_r}$,

$r>b$

$E = 0$

$Q_{enc} = \pi a^2 l \rho$

b) $r<a$

$B = \frac{\mu_0 I_{enc}}{2\pi r} = \boxed{\frac{\mu_0 r \rho \cdot \vec{v}}{2}}$

$I_{enc} = \pi r^2 \rho \vec{v}$

$a<r<b$

$B = \frac{\mu_0 I_{enc}}{2\pi r} = \boxed{\frac{\mu_0 a^2 \rho \vec{v}}{2r}}$

$r>b$

$\boxed{B = 0}$

Q2 $E(z,t) = (E_1 \hat{x} + E_2 \hat{y}) e^{i(kz-\omega t)}$

a) $\nabla \times E = -\frac{\partial B}{\partial t} \Rightarrow B = \int \left[E_2 (ik) e^{i(kz-\omega t)} \hat{x} - E_1 (ik) e^{i(kz-\omega t)} \hat{y} \right] dt.$

$\nabla \times E = -\frac{\partial E_y}{\partial z} \hat{x} + \frac{\partial E_x}{\partial z} \hat{y} + \left(\cancel{\frac{\partial E_y}{\partial x}}^{0} - \cancel{\frac{\partial E_x}{\partial y}}^{0} \right) \hat{z} = -E_2 \cdot (ik) e^{i(kz-\omega t)} \hat{x} + E_1 (ik) e^{i(kz-\omega t)} \hat{y}$

$$ik\left[\frac{-E_2 \cdot e^{i(kz-wt)}}{iw}\hat{x} + \frac{E_1 e^{i(kz-wt)}}{iw}\hat{y}\right] = B$$

b) $E \cdot B = 0 \quad \perp$

$$\left[(E_1\hat{x} + E_2\hat{y})\cdot e^{i(kz-wt)}\right] \cdot \left[(E_1\hat{y} - E_2\hat{x})\cdot\frac{K}{w}e^{i(kz-wt)}\right] = 0$$

$$-E_1E_2\frac{K}{w}e^{2i(kz-wt)} + E_1E_2\frac{K}{w}e^{2i(kz-wt)} = 0$$

$$0 = 0 \checkmark$$

c) $S = E \times H$

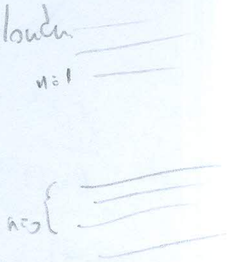
$$= \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ Ex & Ey & 0 \\ Hx & Hy & 0 \end{vmatrix} = (ExHy - EyHx)\hat{z} = E_1^2\frac{K}{w}e^{2i(kz-wt)} - E_2^2\frac{K}{w}e^{2i(kz-wt)}$$

$$(E_1^2 - E_2^2)$$

Q3. energia é um oscilador quântico deslocado

$$\left(v + \frac{1}{2}\right)\hbar w + S_r h l(l+1)$$

n



Q4. a) $(pc)^2 + m_0c^2 = 3\sqrt{(p_1c^2 + m_0c^2)^2}$

$$\frac{h}{\lambda} = 3\frac{h}{\lambda_1}$$

b) $E = 2\gamma m_0 c$

pc

c) $\frac{p^2c^2}{E^2} = 1 - m_0^2c^4$

?

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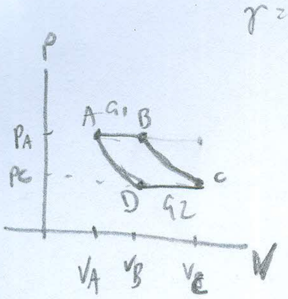
Q5.

$\gamma = \frac{C_p}{C_v}$

a)



b) $A \to B \Rightarrow$

$Q = nC\Delta T + nR\Delta T$

$Q = n(C_v + R)\Delta T$

$\boxed{Q_1 = nC_p\Delta T}$

? $V_A\, P_A\, V_B$

$Q = T \cdot S \Rightarrow Q = \Delta U + W$

$W = p \cdot V = nRT$

$p = \frac{nRT}{V}$ $\Delta U = nC_v\Delta T$

$B \to C$: $\boxed{Q = 0}$

$C \to D$: $\boxed{Q_2 = nC_p\Delta T}$

$D \to A$: $\boxed{Q = 0}$

c) $W = \int_A^B p\,dv + \int_B^C p\,dv + \int_C^D p\,dv + \int_D^A p\,dv = p(V_B - V_A) + nRT\ln\left(\frac{V_C}{V_B}\right) + \ldots$

$\ldots + p(V_B - V_C) + nRT\ln\left(\frac{V_A}{V_B}\right)$

$\eta = \frac{W}{Q} = 1 - \frac{T_1}{T_2} = 1 - \left(\frac{P_1}{P_2}\right)^{\frac{(\gamma-1)}{\gamma}}$

$\Delta U = 0$ because U is a function of state, and any cycle returns the system to its starting state.

$\Delta U = 0 = q_1 + q_2 - W \Rightarrow \boxed{W = q_1 + q_2}$

$q_1 = c_p(T_b - T_a)$

$q_2 = c_p(T_d - T_c)$

$P_c = P_d$; $P_a = P_b \Rightarrow \frac{P_a}{P_c} = \frac{P_b}{P_d}$

$W = c_p[(T_b - T_a) + (T_d - T_c)]$

$\eta = \frac{\cancel{c_p}[(T_b - T_a) + (T_d - T_c)]}{\cancel{c_p}(T_b - T_a)} = 1 + \frac{(T_d - T_c)}{(T_b - T_a)} = 1 - \frac{T_c\left(\frac{T_d}{T_c} - 1\right)}{T_a\left(\frac{T_b}{T_a} - 1\right)} = \boxed{1 - \frac{T_c}{T_a}}$

heat in

Q6. equilibrio $F = -m\omega^2 x$ $\omega^2 = \frac{k}{m}$

a) $F_C = -kx$

$x = x_0 \cdots$

$\frac{c^2}{4x_0^2} = -m\omega^2 x \Rightarrow x_0^2 = \frac{-c^2}{4m\omega^2 x} \Rightarrow x_0^3 = \frac{-c^2}{4m\omega^2}$?

b) $m\ddot{x}_1 = -kx_2' - \frac{c^2}{(x_1-x_2)^2}$ I think $x_i = x_i' + x_0$

$m\ddot{x}_2 = -kx_2' - \frac{c^2}{(x_1-x_2)^2}$ isola x_1'' na 2ª eq e poe na 1ª

$\omega^2 = \frac{k}{m}$

Q8. $|\psi\rangle = \alpha\left(|z_+\rangle - \frac{\sqrt{2}}{2}|z_-\rangle\right)$

a) $|\langle\psi|\psi\rangle|^2 = 1$

$\alpha^2 \left[\langle z_+|z_+\rangle + \frac{1}{2}\langle z_-|z_-\rangle \right] = 1$

$\frac{3\alpha^2}{2} = 1 \Rightarrow \boxed{\alpha = \sqrt{\frac{2}{3}}}$

$|x_+\rangle = \frac{1}{\sqrt{2}}\left(|z_+\rangle + |z_-\rangle\right)$

normalizá para $\langle\psi|\psi\rangle = 1$

b) $\boxed{\langle z_+|\psi\rangle} = \alpha\frac{\sqrt{2}}{2}$

$\boxed{P\left(\frac{\hbar}{2}\right) = \alpha^2 = \frac{1}{3}}$ $\frac{1}{\sqrt{2}}\left(\alpha + \alpha\frac{\sqrt{2}}{2}\right)$

$|\langle z_+|\psi\rangle|^2$

c) $\boxed{\langle x_+|\psi\rangle}$ $\left|\left(\frac{1}{\sqrt{2}} \; \frac{1}{\sqrt{2}}\right)\begin{bmatrix}\alpha \\ \alpha\frac{\sqrt{2}}{2}\end{bmatrix}\right|^2$

$= \frac{1}{\sqrt{2}}\alpha\left(\langle z_+| + \langle z_-|\right)\cdot\left(|z_+\rangle - \frac{\sqrt{2}}{2}|z_-\rangle\right)$

$= \frac{1}{\sqrt{3}}\left(1 - \frac{\sqrt{2}}{2}\right) = \frac{2-\sqrt{2}}{\sqrt{3}\cdot 2}$ ✓

$P\left(\frac{\hbar}{2}\right) = \frac{4+2-4\sqrt{2}}{12} = \boxed{\frac{3-2\sqrt{2}}{6}}$

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Expectation value $\langle S_z \rangle = P\left(S_z = \frac{\hbar}{2}\right)\cdot\frac{\hbar}{2} + P\left(S_z = -\frac{\hbar}{2}\right)\left(-\frac{\hbar}{2}\right) = \frac{\hbar}{2}\cdot\left(\frac{1}{3}\right) = \frac{\hbar}{6}$

$\langle S_x \rangle = P\left(S_x = \frac{\hbar}{2}\right)\cdot\frac{\hbar}{2} + P\left(S_x = -\frac{\hbar}{2}\right)\cdot\left(-\frac{\hbar}{2}\right)$

d) $\vec{S}\hat{n} = \frac{\hbar}{2}\begin{pmatrix} \cos\phi & \sin\phi\, e^{-i\theta} \\ \sin\phi\, e^{i\theta} & -\cos\phi \end{pmatrix} \Rightarrow \phi = 0,\ \theta = 45° \Rightarrow \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ z-axis

$\phi = 90°, \theta = 0 \Rightarrow \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ x-axis

$\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\phi = 45°, \theta = 0 \Rightarrow \frac{\hbar}{2}\begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{pmatrix} = \frac{\sqrt{2}\hbar}{4}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \frac{1}{\sqrt{2}}($

$\begin{vmatrix} \frac{\sqrt{2}}{2} - \lambda & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} - \lambda \end{vmatrix} = 0 \Rightarrow \lambda^2 - \frac{1}{2} - \frac{1}{2} = 0$

$\lambda = \pm 1$

$\lambda = 1$

$\begin{bmatrix} \frac{\sqrt{2}}{2} - 1 & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} - 1 \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = 0 \Rightarrow \alpha\left(\frac{\sqrt{2}}{2} - 1\right) + \frac{\sqrt{2}}{2}\beta = 0 \qquad \begin{bmatrix} 1 \\ 1 - \sqrt{2} \end{bmatrix}$

$\frac{\sqrt{2} - 2}{2}$

$\beta = \frac{\sqrt{2} - 2}{\sqrt{2}} = 1 - \sqrt{2}$

$|S_+\rangle = a\left(|z_+\rangle + (1 - \sqrt{2})|z_-\rangle\right) \qquad |\langle S_+|\psi\rangle|^2$

$a^2(1 + 2 - 2\sqrt{2} + 1) = 1 \qquad |\langle S_-|\psi\rangle|^2 \Rightarrow \boxed{\langle S_+\rangle = |\langle S_+|\psi\rangle|^2\cdot\frac{\hbar}{2} + |\langle S_-|\psi\rangle|^2\cdot\left(-\frac{\hbar}{2}\right)}$

$a = \sqrt{\frac{1}{4 - 2\sqrt{2}}}$

Q9. $[\hat{A}, \hat{H}] \neq 0$

a) $U_+ = \frac{\sqrt{2}}{2}(\phi_+ + \phi_-) = \frac{1}{\sqrt{2}}(\phi_+ + \phi_-)$

$U_- = \phi_+ - \phi_+ \Rightarrow \frac{1}{\sqrt{2}}(\phi_+ - \phi_-)$

$\langle \hat{A} \rangle_{\phi_+} = \langle \phi_+ | \hat{A} | \phi_+ \rangle = \langle \phi_+ | a_+ \phi_+ \rangle = a_+ \langle \phi_+ | \phi_+ \rangle = a_+$

b) Projeção Ψ em $\phi \Rightarrow |\phi\rangle \langle \phi | \Psi \rangle$

$P_{U_-} | H U_+ \rangle = |U_-\rangle \cdot \langle U_- | \hat{H} U_+ \rangle = |U_-\rangle \cdot E \langle U_- | U_+ \rangle = 0$

ortogonais

$\frac{1}{2}(\langle \phi_+ | - \langle \phi_- |)(|\phi_+\rangle + |\phi_-\rangle) = 0$

c) ? Suppose $\Psi(x,t) = \Psi(x) \cdot f(t)$

$i\hbar \frac{df(t)}{dt} \cdot \Psi(x) = \hat{H} \Psi(x) \cdot f(t)$

$f(t) = f(0) \cdot e^{-i\frac{\hat{H}t}{\hbar}} \Rightarrow \Psi(t) = \Psi(0) \cdot e^{-i\frac{\hat{H}t}{\hbar}}$ estranho...?

d) $\langle \Psi(t) | \hat{A} | \Psi(t) \rangle =$

$3\hbar\omega\left(\frac{1}{2} + \frac{1}{\frac{\hbar\omega}{kT}}\right)$

$\hat{=} 3kT$ $\frac{1}{2}$...

$C_V = \frac{\partial E}{\partial T}$

Q10.

a) $H = (n_1 + n_2 + n_3 + \frac{3}{2})\hbar\omega \rightarrow$ for 1D. $Z = e^{-\frac{\beta\hbar\omega}{2}} \cdot \sum e^{-\frac{n\hbar\omega}{\beta}} = \frac{e^{-\frac{\beta\hbar\omega}{2}}}{(1 - e^{-\beta\hbar\omega})}$

$\frac{\hbar\omega}{kT} \ll 1$

$Z = \sum_n e^{-\beta E_n} \Rightarrow Z_{3d}^N = (Z_{1d})^{3N}$

b) $-3 \cdot \frac{\partial}{\partial\beta} \ln Z = -3\left(-\frac{\hbar\omega}{2} - \frac{\hbar\omega e^{-\beta\hbar\omega}}{(1 - e^{-\beta\hbar\omega})}\right) = 3\hbar\omega\left(\frac{1}{2} + \frac{1}{e^{\beta\hbar\omega} - 1}\right)$

$\boxed{6}$